Parallel Component μ_z of Partially Magnetized Microwave Ferrites

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***Abstract*—A formula for the parallel component μ_z of the microwave permeability tensor in the partially magnetized state is derived. The theory is in good agreement with the experimental results.**

I. Introduction

IN THE ACTUAL applications of ferrites, microwave ferrites in the partially magnetized state are very important for the devices such as circulators operating below ferromagnetic resonance, latching phase shifters, and so on. Therefore research on the microwave permeability tensor of partially magnetized ferrites are obviously important. For a magnetic material magnetized in the z-direction, the small-signal microwave permeability is a tensor of the form

$$\overleftrightarrow{\mu}=\begin{pmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_z \end{pmatrix}. \tag{1}$$

The parallel component μ_z is not equal to unity for partially magnetized materials. Using averaging techniques, Rado [1] analyzed the microwave permeability tensor in nonsaturated ferromagnetic materials under the condition that $\omega_M \ll \omega$ and $\omega_e \ll \omega$ (where ω_M, ω_e, and ω are the angular saturation magnetization frequency, the angular effective magnetostatic field frequency, and the angular RF frequency, respectively) and other few restricted conditions. Rado obtained the off-diagonal component κ and parallel component μ_z under these conditions. The parallel component μ_z from Rado's theory was equal to unity for nonsaturated ferromagnetic materials under these conditions and did not apply in the case where $\omega_M \simeq \omega$ and $\omega_e \simeq \omega$, since his formula for μ_z in the partially magnetized state did not agree with the experimental results under the conditions that $\omega_M \simeq \omega$ and $\omega_e \simeq \omega$.

Later, Schleomann [3], [4] presented a formula for the (isotropic) permeability in the completely demagnetized state. When there exists an external dc magnetic field of arbitrary strength, Schleomann's formula cannot be used. The authors [5] presented formulas for the complex transverse components of partially magnetized ferrites derived under some assumptions. The theoretical formulas can represent the experimental values for any strength of the externally applied dc magnetic field with good agreement. Only a few reports are, however, on the parallel component μ_z of partially magnetized ferrites, e.g., [6]–[8]. As far as the authors know there has been only an empirical formula for the parallel component μ_z of partially magnetized ferrites. Green *et al.*, [7], [8] proposed that an empirical equation for the real part of the parallel component μ_z' be given by

$$\mu_z' = \mu_0'^{[1-(M/M_s)^{5/2}]} \tag{2}$$

where μ_0' is the real part of the permeability in the completely demagnetized state [3], [4], M is the magnetization, and M_s is the saturation magnetization.

There are many microwave ferrite devices which operate in the region below saturation. In this region, the real part of the parallel component μ_z' is, as shown later, less than unity, particularly for the completely demagnetized state. The imaginary part μ_z'' has a large value for $\omega_M/\omega \geqq \sqrt{5}/3$ [5]. Thus the attenuation due to μ_z'' can not be neglected. Therefore, it is obviously important to formulate the complex parallel component μ_z. In this paper, using averaging techniques and an adjustable parameter, we first present a formula for the complex parallel component of the microwave permeability tensor in the partially magnetized state. Lastly, the theory will be compared with the experimental results.

II. Formulation for μ_z

If the magnetic inhomogeneities are larger than the domain wall thickness, the exchange torque may be neglected at points which are not contained within walls. Consequently, at such points, the magnetization vector $\vec{M}(t)$ and effective magnetic field vector $\vec{H}_e(t)$ are related by Gilbert's equation [9], [10]

$$\frac{d\vec{M}(t)}{dt} = \gamma \vec{M}(t) \times \vec{H}_e(t) + \frac{\alpha}{M_s} \vec{M}(t) \times \frac{d\vec{M}(t)}{dt} \tag{3}$$

where γ is the gyromagnetic ratio and α the phenomenological loss term.

Furthermore, a time-independent unit vector $\vec{u}$ shows the equilibrium direction of the saturation magnetization vector $\vec{M}_s$ at the point in question. Since the magnetization

Manuscript received May 15, 1980; revised December 18, 1980. A condensed version of this paper has been presented at the 1980 IEEE-MTTS International Microwave Symposium, Washington, DC.

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0018-9480/81/0600-0568$00.75

within the domain is saturated, the unit vector $\vec{u}$ is defined as follows:

$$\vec{M}(t)=M_s\vec{u}+\vec{m}(t) \tag{4}$$

where $\vec{m}(t)$ is the RF magnetization vector. Writing down the RF magnetic field $\vec{h}(t)$, the locally effective magnetic field $\vec{H}_e(t)$ is written as follows:

$$\vec{H}_e(t)=H_e\vec{u}+\vec{h}(t) \tag{5}$$

where $H_e\vec{u}$ is a locally effective magnetostatic field and represents the u-component of the resultant field of the applied field, anisotropy field, and so on.

In small-signal analysis, assume that $m/M_s \ll 1$ and $h/H_e \ll 1$. Calculating a vector product of (3) and $\vec{u}$, and arranging (3) by substituting $\vec{m}\times\vec{u}$ obtained here into (3), we obtain, with the time-dependence of $\exp(j\omega t)$ (Appendix I)

$$\vec{m}=-\left(\frac{\omega_e}{j\omega}+\alpha\right)^2\vec{m}+\frac{\mu_0\omega_M}{j\omega}\left(\frac{\omega_e}{j\omega}+\alpha\right)\left\{\vec{h}-(\vec{h}\cdot\vec{u})\vec{u}\right\} + \frac{\mu_0\omega_M}{j\omega}\vec{h}\times\vec{u} \tag{6}$$

where $\omega_e=-\gamma H_e$, $\omega_M=-\gamma M_s/\mu_0$, and μ_0 is the intrinsic permeability of free space. Letting

$$\vec{u}=\vec{i}\alpha_1+\vec{j}\alpha_2+\vec{k}\alpha_3 \qquad \alpha_1^2+\alpha_2^2+\alpha_3^2=1$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors along the x, y, and z coordinate axes, respectively. From (6), we obtain

$$\vec{m}=\mu_0 \cdot \begin{pmatrix} \chi(1-\alpha_1^2) & -\chi\alpha_1\alpha_2-j\kappa\alpha_3 & -\chi\alpha_1\alpha_3+j\kappa\alpha_2 \\ -\chi\alpha_2\alpha_1+j\kappa\alpha_3 & \chi(1-\alpha_2^2) & -\chi\alpha_2\alpha_3-j\kappa\alpha_1 \\ -\chi\alpha_3\alpha_1-j\kappa\alpha_2 & -\chi\alpha_3\alpha_2+j\kappa\alpha_1 & \chi(1-\alpha_3^2) \end{pmatrix}\vec{h} \tag{7}$$

where

$$\chi=\frac{\omega_M(\omega_e+j\omega\alpha)}{-\omega^2+(\omega_e+j\omega\alpha)^2} \tag{8}$$

$$\kappa=\frac{-\omega\omega_M}{-\omega^2+(\omega_e+j\omega\alpha)^2}. \tag{9}$$

In this paper, we consider the case where the material is magnetized parallel to the z-axis. The parallel component μ_z of the permeability tensor is given as (Appendix II)

$$\mu_z=1+\chi(1-\alpha_3^2). \tag{10}$$

The direction of the domain may be sufficiently random, particularly in a polycrystal, that is,

$$\langle\alpha_1\rangle=0 \qquad \langle\alpha_2\rangle=0 \qquad \langle\alpha_3\rangle=\frac{M}{M_s}$$

$$\langle\alpha_1^2\rangle=\langle\alpha_2^2\rangle=\frac{1}{2}\{1-\langle\alpha_3^2\rangle\} \tag{11}$$

where symbol $\langle\ \rangle$ means a spatial average and M is an average magnetization of the z-direction. Consequently, for partially magnetized materials with random domain orientation, the parallel component of the effective permeability tensor of two or more domains $\langle\mu_z\rangle$ is given by

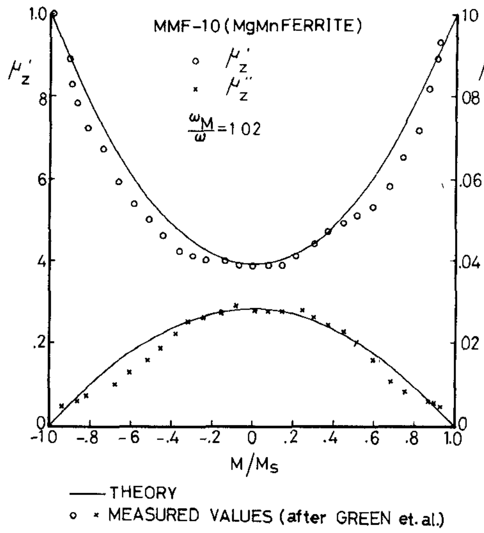

Fig. 1. Real and imaginary parts of parallel component of partially magnetized ferrites as a function of M/M_s. $\omega_e=16.2$ GHz, $\alpha=1.35\times10^{-2}$, $f=5.5$ GHz (Green *et al.* [7]).

$$\langle\mu_z\rangle=1+\frac{\omega_M(\omega_e+j\omega\alpha)}{-\omega^2+(\omega_e+j\omega\alpha)^2}\{1-\langle\alpha_3^2\rangle\}. \tag{12}$$

The parallel component μ_z from Rado's theory is independent of an average magnetization M but is equal to unity under the conditions that $\omega_M \ll \omega$ and $\omega_e \ll \omega$. Consequently, under the conditions that $\omega_M \simeq \omega$ and $\omega_e \simeq \omega$, Rado's μ_z did not agree with the experimental results [6]–[8]. The parallel component $\langle\mu_z\rangle$ in (12) is a quadratic function of α_3. As we increase M, and the magnetization approaches saturation in the z-direction $\langle\alpha_3^2\rangle \to 1$, the real part of $\langle\mu_z\rangle$ approaches 1 but on the other hand the imaginary part of $\langle\mu_z\rangle$ approaches 0. This is physically very reasonable. We, however, do have an adjustable parameter, namely ω_e, even when $\alpha=0$. Equation (12) is not rigorous because we cannot theoretically calculate ω_e. As shown in Section III, however, (12) does hold rigorously if ω_e is obtained from experiment.

III. Comparison of the Experimental Results

In (12) we express $1-\langle\alpha_3^2\rangle$ by approximating it as follows:

$$1-\langle\alpha_3^2\rangle \simeq 1-\langle\alpha_3\rangle^2=1-\left(\frac{M}{M_s}\right)^2. \tag{13}$$

Substituting (13) into (12), the real part and the imaginary part of parallel component $\langle\mu_z\rangle$ are μ_z' and μ_z'', respectively. The formula for the parallel component has been compared to measurements by Green *et al.* [7] of μ_z' and μ_z'' as a function of M/M_s.

In fitting the theoretical formula to the experimental results the parameters ω_e and α have to be suitably chosen, since they cannot be independently determined. Typical results are shown in Figs. 1–5. Comparison of the experimental results and the theory in Figs. 1–5 shows that the

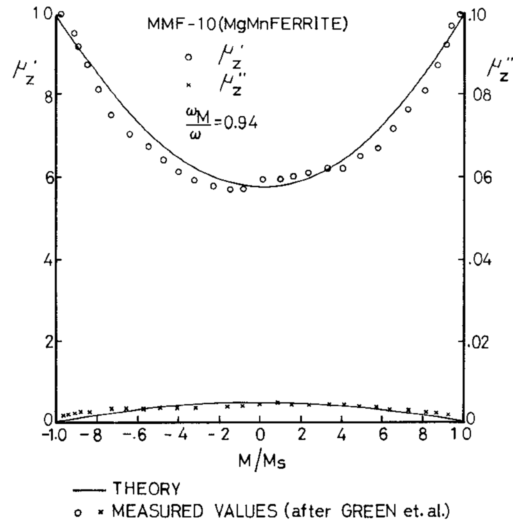

Fig. 2. Real and imaginary parts of parallel component of partially magnetized ferrites as a function of M/M_s. $\omega_e=13.3$ GHz, $\alpha=0.30\times 10^{-2}$, $f=5.5$ GHz (Green *et al.* [7]).

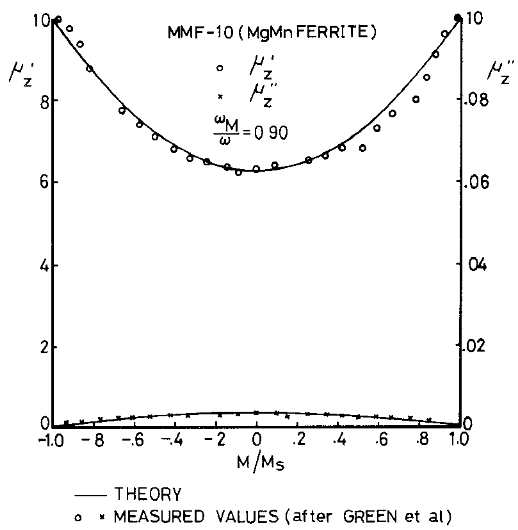

Fig. 3. Real and imaginary parts of parallel component of partially magnetized ferrites as a function of M/M_s. $\omega_e=12.5$ GHz, $\alpha=0.25\times 10^{-2}$, $f=5.5$ GHz (Green *et al.* [7]).

agreement is reasonably good throughout the partially magnetized region $(-1\leq M/M_s\leq 1)$. The value of imaginary part μ_z'' differs slightly from the experimental value, as the absolute value of M/M_s approaches to 1. The value of theoretical formula μ_z becomes equal to 1 at $\langle\alpha_3^2\rangle=1$. This makes little difference between the theory and the experimental results because the imaginary part of μ_z has actually a very small value even if the magnetization became saturation in the z-direction.

IV. Conclusion

A formula for the parallel component of the microwave permeability tensor of partially magnetized materials is for the first time derived as a function of magnetization. The theory is useful for partially magnetized microwave ferrite devices and is in good agreement with the experimental results.

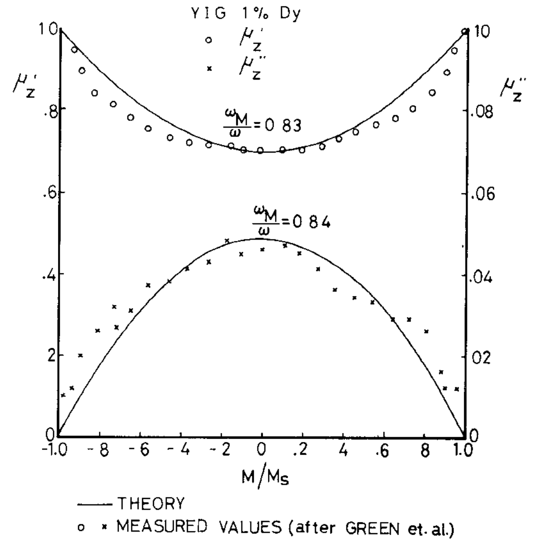

Fig. 4. Real and imaginary parts of parallel component of partially magnetized ferrites as a function of M/M_s. $\omega_M/\omega=0.83$: $\omega_e=11.2$ GHz, $\alpha=0.40\times 10^{-2}$; $\omega_M/\omega=0.84$: $\omega_e=11.4$ GHz, $\alpha=0.41\times 10^{-2}$; $f=5.5$ GHz (Green *et al.* [7]).

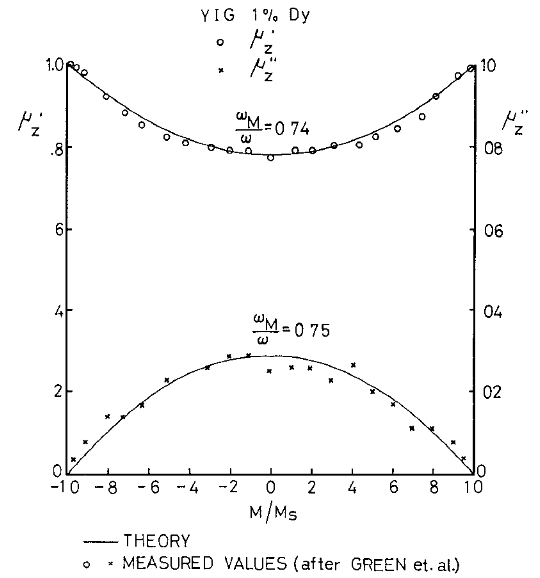

Fig. 5. Real and imaginary parts of parallel component of partially magnetized ferrites as a function of M/M_s. $\omega_M/\omega=0.74$: $\omega_e=9.5$ GHz, $\alpha=0.29\times 10^{-2}$; $\omega_M/\omega=0.75$: $\omega_e=9.6$ GHz, $\alpha=0.30\times 10^{-2}$; $f=5.5$ GHz (Green *et al.* [7]).

Appendix I

Assuming that $m/M_s\ll 1$ and $h/H_e\ll 1$, we obtain, with the time-dependence of $\exp(j\omega t)$

$$\vec{m}=-\left(\frac{\omega_e}{j\omega}+\alpha\right)\vec{m}\times\vec{u}+\frac{\mu_0\omega_M}{j\omega}\vec{h}\times\vec{u}. \tag{A1}$$

From a scalar product of (A1) and $\vec{u}$, we obtain

$$\vec{m}\cdot\vec{u}=0. \tag{A2}$$

Substituting (A2) into $(\vec{m}\times\vec{u})\times\vec{u}$, we obtain

$$(\vec{m}\times\vec{u})\times\vec{u}=(\vec{m}\cdot\vec{u})\vec{u}-(\vec{u}\cdot\vec{u})\vec{m}$$
$$=-\vec{m}. \tag{A3}$$

Therefore, from a vector product of (A1) and $\vec{u}$, we obtain

$$\vec{m}\times\vec{u}=\left(\frac{\omega_e}{j\omega}+\alpha\right)\vec{m}-\frac{\mu_0\omega_M}{j\omega}\{\vec{h}-(\vec{h}\cdot\vec{u})\vec{u}\}. \quad \text{(A4)}$$

Consequently, we obtain (6) by substituting (A4) into (A1).

Appendix II

For the case where the direction of the domain is sufficiently random

$$\langle\alpha_i\alpha_j\rangle=0, \qquad i\neq j. \quad \text{(A5)}$$

Consequently, for partially magnetized materials with random domain orientation, the effective permeability tensor of two or more domains is given by

$$\vec{\mu}=(\langle\mu_{ij}\rangle)$$
$$=\begin{pmatrix} 1+\chi\{1-\langle\alpha_1^2\rangle\} & -j\kappa\langle\alpha_3\rangle & 0 \\ j\kappa\langle\alpha_3\rangle & 1+\chi\{1-\langle\alpha_2^2\rangle\} & 0 \\ 0 & 0 & 1+\chi\{1-\langle\alpha_3^2\rangle\} \end{pmatrix} \quad \text{(A6)}$$

where $i=1,2,3$ and $j=1,2,3$.

The off-diagonal components $\langle\mu_{ij}\rangle (i\neq j)$ in (A6) are in agreement with the form as Rado's theory [1], [2]. However, the diagonal components $\langle\mu_{11}\rangle$, $\langle\mu_{22}\rangle$ of the transverse permeability components are not in agreement with the experimental results [5], [7], [8] except the case where the materials are magnetically saturated ($\langle\alpha_1^2\rangle=0$, $\langle\alpha_2^2\rangle=0$). At present, unfortunately, the authors have no better justification for the formulas than that presented in this paper, but further investigations are in progress.

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